

GFT Summary

What is the GFT? GFT stands for General Feedback Theorem. It is a mathematical model of a control system as illustrated in Figure 1 and Figure 2. Dr. R. David Middlebrook with the California Institute of Technology developed the theorem, which describes a control system in terms of its response to signal injection, usually in the principal forward control loop. It arises from application of a bi-linear transform, originally described by Bode in 1945, that gives the sensitivities of a driving point or transfer impedance at one set of terminals with respect to an impedance elsewhere in the mesh. Later, Middlebrook extended Bode's work with his Extra Element Theorem (EET), then successively by applying the EET within a control loop using voltage and current sources instead of impedances.

How is it used? The GFT can be invoked to measure open loop control system characteristics while the loop is closed, thereby preserving the correct loading and DC bias. The principal loop gain is the loop gain commonly referred to in a Bode plot and it is used to determine the stability margin of a control system. Breaking the loop to measure this open loop gain disturbs the loop, removing the natural loading at the cut point; moreover, the DC operating point is difficult to establish with the loop open. For both simulation and hardware measurement, the DC loop can be left closed by using a large series inductor and injecting the test signal using a coupling capacitor, but this approach ignores circuit loading. The GFT uses dual voltage and current injection under the following conditions to measure the loop gain, T.

$$\frac{1}{T} = \left. \frac{ix}{iy} \right|_{vy=0} + \left. \frac{vx}{vy} \right|_{iy=0} + \left. \frac{ix}{iy} \right|_{vx=0} \left. \frac{vx}{vy} \right|_{iy=0}$$

Note: This form, using x/y terms instead of 1/(y/x) is preferred because y/x is frequently infinite.

A constant voltage injection is used while the current injection amplitude is adjusted to create a null condition for v_y , i_y and v_x , as shown in Figure 3. In hardware this would be achieved using a complex signal (sine and cosine generators) to force the complex null conditions. For simulation, the null result is achieved by placing a high gain amplifier from the voltage or current to be nulled to the current injection source. Intusoft Newsletter #69 provides the mathematical foundation for using this technique.

ICAP/4 Windows has incorporated GFT models and GFT scripts to accomplish this task and more.

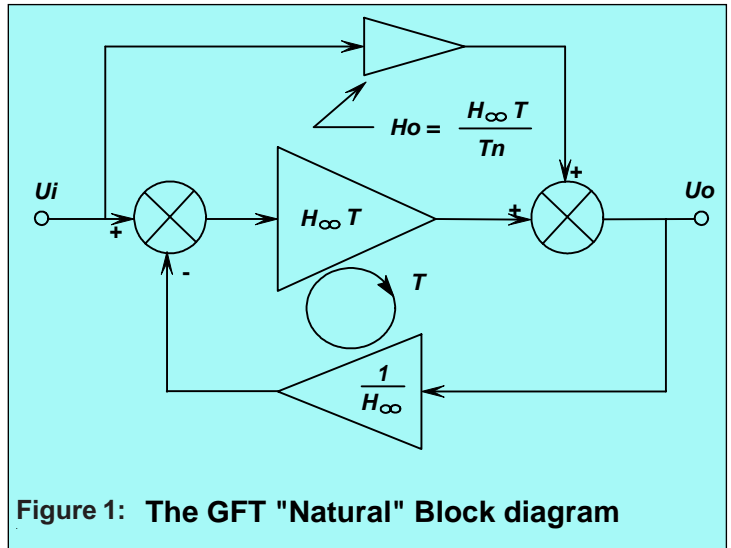


Figure 1: The GFT "Natural" Block diagram

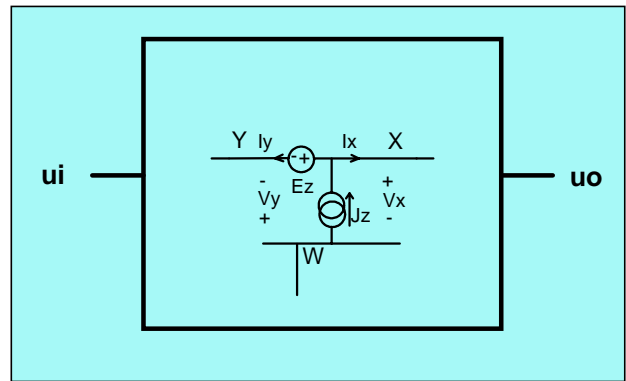


Figure 2: Injecting current and voltage within the principal loop

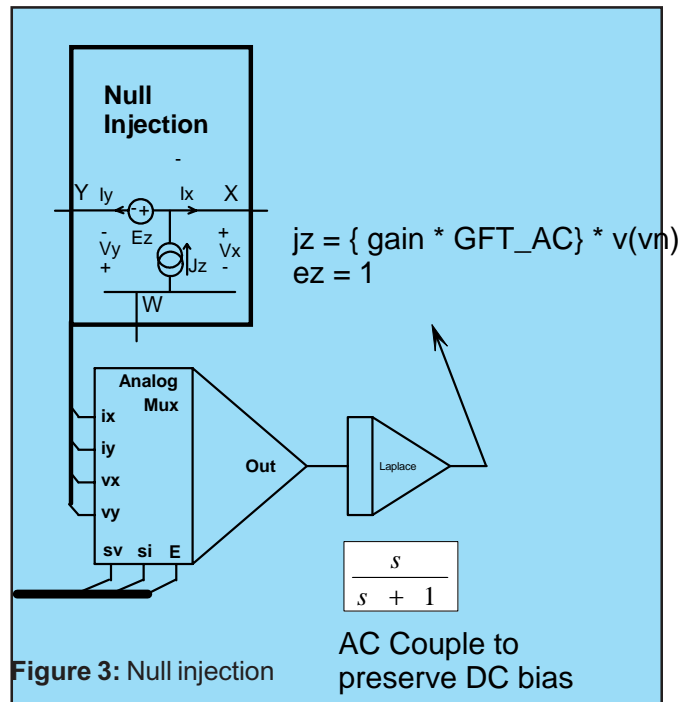


Figure 3: Null injection

Is there something else the GFT can be used for? The GFT can also be used as a top-down design tool. By making the complete set of null injection measurements, all of the control system properties are easily exposed. This not only applies to the loop stability, but the conformance of the closed loop to the designers specification. ICAP/4 Windows provides ALL combinations of single and dual injection with 28 models; 5 GFT simulation templates; and an in depth manual with several examples - written by Dr. Middlebrook - that explains how the system works. You can learn more by going to www.intusoft.com/gft.htm.

Are there limitations? Yes, the parameters v_y , i_y and v_x must respond to the injection current J_z and voltage E_z . That means the y terminal can be neither a hard voltage or current source. Also, the x terminal cannot be a voltage source. Inserting a small series resistance or a large shunt resistance overcomes the problem. Single injection models are also available, which overcome these restrictions. For the more general case, the injection point must be in the principal forward loop to acquire all of the data. Selection of the injection point is critical for acquiring crucial design information. For in-depth discussions of the design process read "The general feedback theorem: a final solution for feedback systems." IEEE Microwave Magazine, April 2006, pages 50-63, available at <http://resolver.caltech.edu/Caltechauthors:MIDieemm06>.

What kinds of circuits need this technique? Two example circuit types requiring GFT are noteworthy. A hysteretic switched mode power supply is one example and RF amplifier is the other. For the later case, you can reference the GFT manual at www.intusoft.com/gft.htm.

Switched Mode Power Supply: Many SMPS circuits do not have an appropriate place to break the loop because the control circuitry is inaccessible. An Average model can provide some access to the internal signals using a simulator. However the Average model must be validated using either time domain simulation methods or applying a hardware transfer function analyzer. The first method is explored in Figure 5 and contains an ON Semiconductor NCP105X series

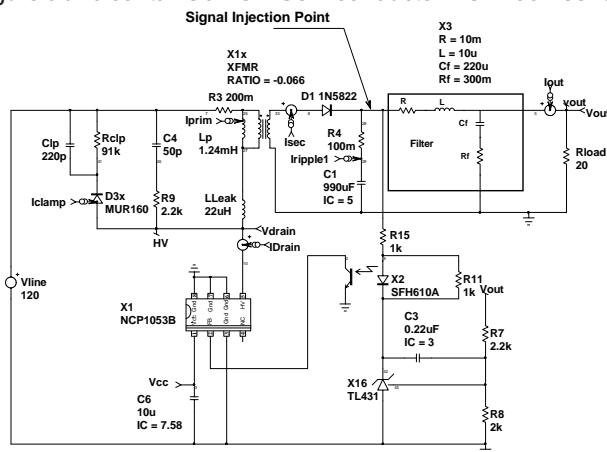


Figure 5 NCP1053B Flyback Application

hysteretic converter. The power of using a GFT injection point is used to achieve our goal. Shown is a self-oscillating circuit that exhibits sub-harmonic oscillation about a mean carrier frequency. Figure 6 shows the power spectral density without a test signal. Then, applying a 100mv test signal at 400Hz results in the Waveforms shown in Figure 7 from a transient GFT simulation. The simulation time for a single frequency was over 10,000 seconds and generated four million-point waveforms. Figure 8 is the result of an AC analysis using an Average model, requiring only 4 seconds to obtain 400 data points.

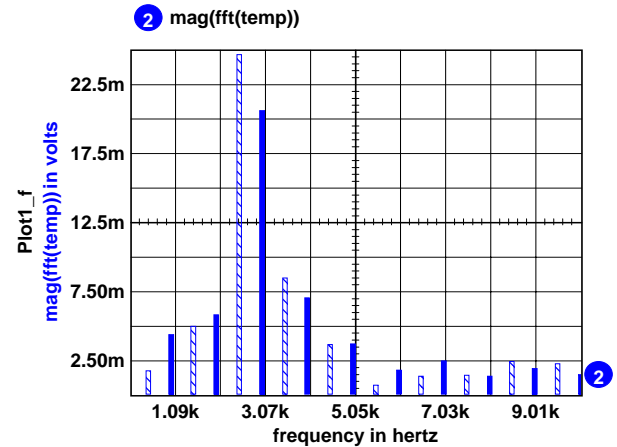


Figure 6: Power Spectral Density shows noise spread about the average switching frequency.

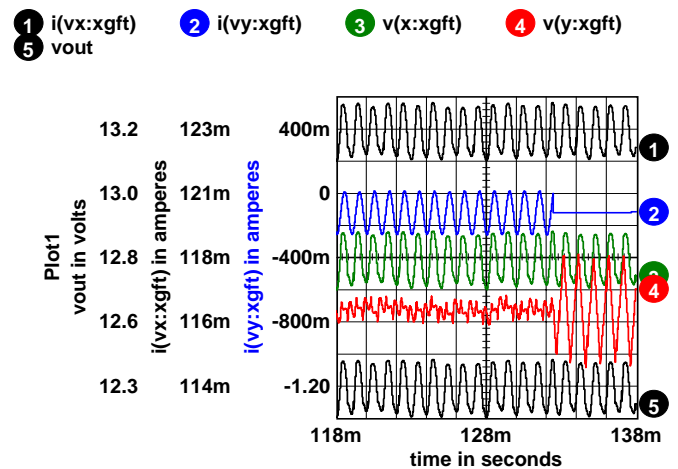


Figure 7: The GFT applied in the time domain

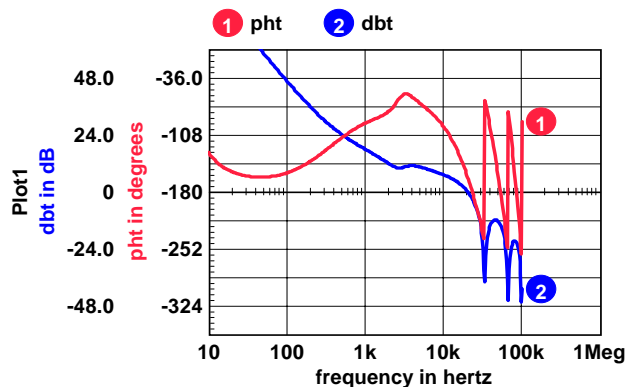


Figure 8: The GFT used with the Average model shows small phase margin between 10 and 100Hz